Multi-Level Ant Colony Algorithm for Optimal Placement of Capacitors in Distribution Systems

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***Abstract*-In this paper, an ant colony algorithm is proposed to determine the optimal locations and ratings of capacitors in a distribution network for reactive power compensation. The approach is multilevel. Two separate tables of pheromones are maintained by the algorithm. Ants generate solution stochastically, based on these pheromone tables. The pheromone tables are updated periodically, so that pheromones accrue more along better solutions. Results obtained by the proposed algorithm have been compared with earlier schemes. We conclude that the proposed approach is an effective approach for optimally placing capacitors in a distribution system.**

I. INTRODUCTION

Capacitors are commonly used in distribution networks for reactive power compensation. They reduce the transmission & distribution losses and improve the over all power factor of the system whose effect is seen in terms on higher returns in revenue metering. An indirect advantage of capacitor placement is seen as a reduction in the amount of VAR transportation required at higher voltage levels on transmission systems. They also help in maintaining the voltage profile within acceptable limits. The amount of benefit that can be reaped by placing the capacitors depend mainly on how the capacitors are placed on the distribution system.

The problem of capacitor placement in a distribution system consists of finding the sizes, location and the number of capacitors that have to be placed on the network. This is one of the combinatorial optimization problems with the size of search space is being equal to $(k+1)^n$, where n is the number of buses on the distribution system and k is the number of possible capacitor sizes that can be placed on the network. This problem has traditionally been solved using mathematical programming techniques. Grainger and Lee formulated the problem as a nonlinear programming model with both the capacitor sizes and location as continuous variables [2]. Given the amount of computational effort involved in solving this problem, artificial intelligence techniques can lead to solutions very close to the optimal solution with only a fraction of the computational effort required for a mathematical programming technique.

Sundararajan and Pahwa have proposed a solution to the capacitor problem using a genetic algorithm approach [1]. Kyu-ho Kim et al considered the unbalance in the system and solved the problem, again using genetic algorithms to search through the solution space [3,4]. Ghose *et al* used the hybrid genetic algorithm- simulated anealing technique for solving the problem[6]. CS Chang and others used the Tabu search strategy and modeled the objective function as a non-differential savings function [5].

Ant colony optimization, which is based on the foraging behavior of ants, has been successfully employed to solve many classic combinatorial optimization problems like the travelling salesman problem, quadratic assignment problem and the network routing problem[7,9,11,12]. Given its efficiency in solving the discrete optimization problems it can be very helpful in solving the problem of optimal placement of capacitors, which can be modeled as a discrete problem.

In this paper we present a methodology for employing the ant colony optimization to the capacitor selection problem. We assume that the system is a balanced three-phase distribution network. For the sake of simplification the operation and maintenance costs of the capacitors have been ignored while modeling the problem. The next sections of the paper discuss the formulation of the capacitor placement problem, the ant colony algorithm and how the ant colony optimization technique can be employed to solve it. This method was tested on a practical radial distribution network with 30 buses and the results have been presented.

II. PROBLEM FORMULATION

While formulating the problem, the load variations have been taken into account. The load duration curve has been approximated as a piece wise linear function by assuming that all the loads vary in a conformal way and the load variations are in discrete levels.

The objective function involving various cost components is given as,

$$Cost = K_e \sum_{i=1}^{L} T_i P_i + K_p P_0 + K_c \sum_{j=1}^{n} C_j . \qquad (1)$$

where K_e, K_p, K_c are constants, referred to as the energy, peak power and the capacitor cost constants.

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The first term on the right hand side of (1) pertains to the cost arising with different load levels in the system. Although the loads can acquire any continuous value, (depending on demand and time of day), we have, for simplicity, assumed L discrete load levels as discussed earlier. The energy consumed in each load level i is a product, P_iT_i, where P_i is the power loss at the load level i and T_i is the time duration for which the load persists in the system. Although not explicitly shown, the peak power depends on the capacitors that have been placed in the feeder's buses, and to compute them requires computationally intensive load flow calculations. The cost component associated with second term, which is linearly related to P_0 is the peak power loss. The last term involves the cost of the capacitors. As mentioned previously, the operation and maintenance costs of the capacitor banks have been ignored and only the fixed purchase and installation costs have been considered. Hence we assume that the capacitor cost is proportional to its KVA rating C_j.

III. ANT COLONY OPTIMIZATION

The ant colony approach is a stochastic algorithm for combinatorial optimization that is derived from the foraging behavior of ants. Despite their extremely simplistic behavior, ants in the real world cooperate to obtain an optimal path from their nest to a food source. This is accomplished by means of special secretions called pheromones, which enables the ants to interact with one another. Pheromones are deposited along the trails taken by the ants. They also evaporate uniformly over time. Eventually, pheromones accumulate more along the optimal trails.

Ant colony algorithms make use of the tables of pheromones to record the optimality of assigning particular values to the various components that make up a solution [8,9]. The entries of the pheromone tables are updated periodically based on the cost of the solutions. When the cost of a solution is low, the corresponding entries in pheromone tables are incremented by a larger amount. The ant colony algorithm was originally proposed as a method for the traveling salesperson problem, where the problem is also one of determining a minimum distance path. Subsequently it has been also shown to work well for other complex optimization problems [10,11].

In each iteration of the algorithm, new solutions are generated. Although the search process is random, the algorithm is biased towards assigning values to the individual components of a solution, which have higher concentrations of pheromones in the pheromone tables. In this manner, the search is restricted to better regions in the solution space. Theoretical investigations have shown that ant colony algorithms are equivalent to gradient descent within a multidimensional pheromone space [14].

As pheromones accumulate along better components, the algorithm produces solutions with decreasing costs, until it is terminated. The lowest cost solution is kept aside and upon termination, becomes the final output of the algorithm. It has been shown that mutating the pheromone tables by adding small random perturbations improves the search by preventing premature convergence to locally optimal solutions [13].

In this paper, the ant colony algorithm for the optimal capacitor placement problem is described. Because of the nature of the problem, the proposed algorithm maintains pheromone tables at two distinct levels that are used by the ants to build solutions. Because of this hierarchy, the approach will be referred to as a multilevel ant colony algorithm.

The proposed algorithm creates a new solution by placing a capacitor at a few of the buses. Unlike in standard combinatorial optimization problems, a two-stage decision is required in deciding the placement of the capacitors. First, because of various problems associated with installation and maintenance, power companies prefer to place capacitors in only a few of the locations. Hence a set of optimal buses has to be identified within the feeder where capacitors have to be placed. The second decision is to decide on the capacitors rating in KVA that would minimize the total cost. Since commercial capacitors are available only of fixed ratings (usually 300, 600, 900, or 1200 KVA), a discrete decision has to be made here too. In order to enable the algorithm simulate this process, we have designed an ant colony algorithm that maintains two pheromone tables.

In the proposed multilevel approach, the first pheromone table is a $1Xn$ array of positive numbers, T_1. Henceforth, we will use the subscripts '1' and '2' to denote the level in some of the algorithm parameters. Using this upper level table, and the total number of capacitors to be placed, m, which is maintained at a constant value in the algorithm, the locations of those capacitors are decided in a stochastic manner. A total of m locations are identified by the algorithm one location at a time. The probability of a capacitor being placed in each time step in bus location j, $p_1(j)$, is given by the following equation,

$$p_1(j) = \frac{T_1(j)}{\sum_{k=1}^{n} T_1(k)} \tag{2}$$

In the above equation, the summation is carried out over all n buses where capacitors can be installed. Thus the probability of a capacitor being installed at any location j, is proportional to the pheromone deposition in the corresponding entry of the upper level pheromone table. The standard roulette wheel method of selection is implemented to simulate the random process. The bus locations selected through this process will be denoted as $j_1, j_2, \ldots j_m$.

The next step is to decide the KVA ratings at the locations j_1, j_2, ... j_m. This is done at the second level, making use of another table of pheromones. This pheromone table is arranged in the form of a rXn matrix T_2, where R is the number of discrete KVA ratings. The probability of assigning a KVA rating corresponding to the row r of the pheromone table to the capacitor at each location j_k, $p_2(r, j_k)$ is equal to,

$$p_2(r, j_k) = \frac{T_2(r, j_k)}{\sum_{r=1}^{R} T_2(r, j_k)} \tag{3}$$

In order to determine the capacitor ratings from the probabilities, another roulette wheel selection had to be performed.

Following the two-stage preliminary assignment of capacitors to the buses, a local search technique is invoked to improve the placement further. To each bus j_k where a capacitor is placed, if the KVA rating selected is the one that corresponds to row r of T_2, then the ratings associated with the adjacent rows r-1 and r+1 are examined. The costs of all three possibilities are computed using numerical load flow simulations, and the rating that produces the minimum cost (see (1)) is chosen to be the final assignment. Because of the significant amount of computation involved to carry out the simulations, only one pass is allowed for each location, in order of their appearance in the system.

The algorithm records the best solution obtained since the first iteration. This solution will be referred to as the global best solution. The final placement cost at the end of any given iteration is compared with the global best, and if it is lower than the latter, the global best is updated accordingly. When the global best does not improve over a period of time (λ iterations), the pheromone tables are perturbed to allow the search to explore further by an operator called mutation [13]. The mutation operator adds a randomly generated value to every entry in the tables. The pheromone tables at both levels are perturbed according to the following,

$$T_1(j) = T_1(j) + \zeta U(-1,1) \tag{4}$$

and

$$T_2(r, j_k) = T_2(r, j_k) + \xi U(-1,1) \tag{5}$$

where ζ and ξ are two small parameters, and $U(-1,1)$ is a uniformly distributed random number in the range [-1, 1].

The pheromone tables are also updated to allow more pheromones to accrue along optimal entries. The amount of pheromones deposited is inversely proportional to the cost of the global best, $\cos t_{gb}$. and is carried out whenever a new global best is found. When j is a bus location selected for a capacitor corresponding to row r in T_2 to be installed, this updating is done as,

$$T_1(j) = (1-\rho_1)T_1(j) + \rho_1 \frac{Q_1}{\cos t_{gb}} \tag{6}$$

and

$$T_2(r, j) = (1-\rho_2)T_2(r, j) + \rho_2 \frac{Q_2}{\cos t_{gb}} \tag{7}$$

where ρ_1 and ρ_2 are two ant colony parameters called the evaporation rates, while Q_1 and Q_2 are two other constants associated with pheromone deposition. For all other entries in either table, only evaporation is performed as,

$$T_1(j) = (1-\rho_1)T_1(j) \tag{8}$$

and

$$T_2(r, j) = (1-\rho_2)T_2(r, j)\,. \tag{9}$$

The algorithm is run for a significant number of iterations until a good solution is found.

IV. RESULTS

The proposed multi-level ant colony optimization approach has been implemented in MATLAB and run on a Pentium IV, 2.4 GHz processor. The test system is a 30-bus radial distribution system with one main feeder and 6 laterals [1] as shown in figure 3. For the system and load data one is referred to [1,2]. The program uses the Newton-Raphson load flow technique to calculate the cost function given in (1). The following cost figures were adopted during the implementation. Energy cost constant K_e was taken as 30 mills / kwh. The peak power cost and the capacitor cost constants are taken at K_p = \$120 / kw / year and K_c = \$5 / kvar. The capacitor cost is considered at a fixed rate of 15% of the total cost on an annual basis.

Although the ant colony approach looks straight forward, the design of a successful and efficient algorithm requires a very carefully selected and finely tuned set of algorithm parameters. The speed of convergence of the algorithm and the quality of the final solution of any ant colony optimization technique highly depends on the choice of the T, ρ, Q, ζ and ξ values. After various trails and tests, the parameters were set the following values. The entries of both pheromone tables, T_1 and T_2 were initialized to a value of 4. Any higher value would only reduce the sensitivity of the pheromone table to the initial set of ants and delay the convergence. The evaporation rates ρ_1 and ρ_2 was set to 0.012. Since the pheromone table was updated only when a new global best was hit, higher evaporation rates would lead to premature convergence. It was observed that in cases where the evaporation rates were higher than this value, the ants tend to lose their ability to explore the search space properly. This would lead to the sub-optimal convergence of the algorithm. Another crucial parameter Q was taken as 75 X 10^5. The parameters ζ, ξ, α and β were set to 1.0.

The results shown here are based on the average of 20 runs of the algorithm. The pheromone table was mutated for every 400 iterations, which did not result in a new global best value. This resulted in perturbing the pheromone table and started the exploration from a new point in the search space. In most of the cases, it was seen that the pheromone table starts to saturate around 650 to 700 iterations (refer to Figure 1).

The best solution from this algorithm gave a cost function value of $212,377.4. This converts into a cost saving of $40,758 when compared to a bare system without any capacitors. The result of this ant colony optimization technique is $1354 better than the method proposed in [1] which gives an optimized cost function value of $213731.4. This method also outperforms the method proposed in [1], in the number of capacitor placements that are required in the system. Using this method we obtained a lower cost solution with only 9 capacitors in the distribution system in comparison to the 10-capacitor placements that are required from the genetic algorithm method proposed in [1]. This would translate into some additional cost savings from the operation and maintenance cost of the extra capacitor bank installed on the system. These costs have been neglected in the problem formulation for the sake of simplicity.

The genetic algorithm approach discussed in [1] considered a population size of 50 and required on an average, 80 to 100 generations to converge to a solution. This would translate into a computational effort of 4000 to 5000 cost function evaluations. On the contrary, the proposed technique required on an average 1000 cost function evaluations to converge to a good solution.

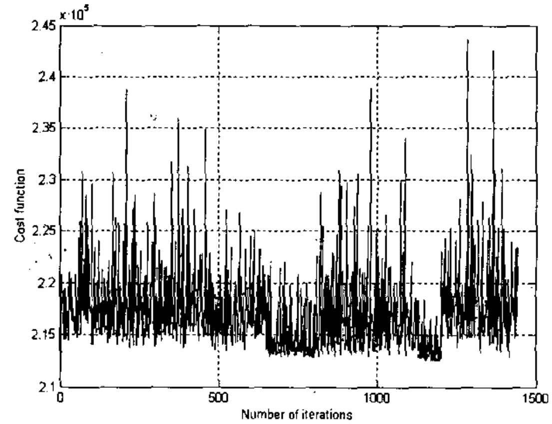

Figure 1: A sample run showing the effect of mutation. It can be clearly noticed that the cost values tend to settle around 750 iterations. However, after the mutation takes place around 800th iteration, the search is again as intense as it is initially. We can also see that the pheromone table undergoes another round of mutation around 1200th iteration.

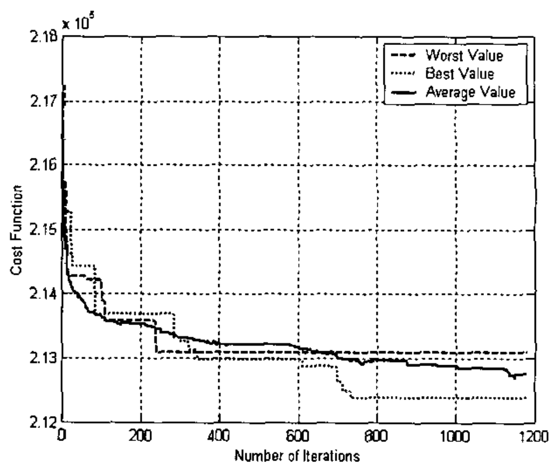

Figure 2: Plot showing the improvement of the global best value as the simulation progresses. Best run, worst run and the average of 20 runs of the algorithm are shown in the figure.

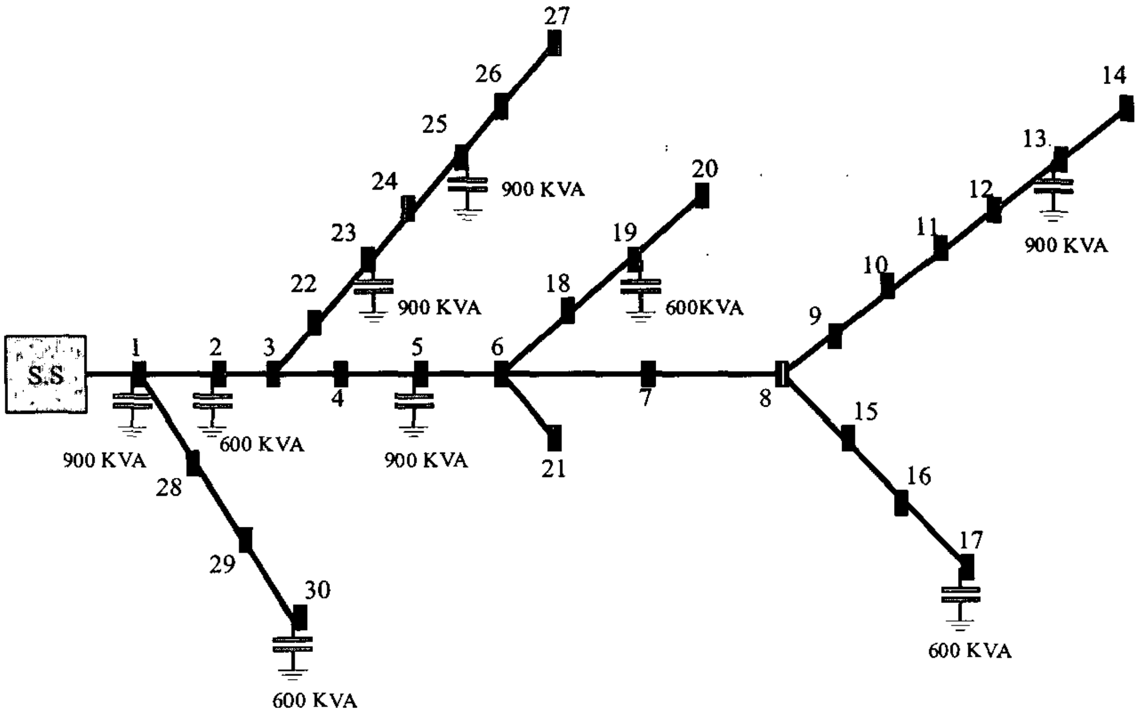

Figure 3: The 30 bus radial distribution system, used as a test case in this work, is shown with the 9 capacitors placed on it.

	Genetic Algorithm	Ant colony optimization
Cost ($)	213,731.4	212377.4
Capacitors required	10	9
Cost function evaluations	~4000	~1000

Table 1: A comparison between the genetic algorithm method adopted in [1] and the ant colony optimization approach.

V. CONCLUSIONS

We have proposed an ant colony approach to determine optimal placement of capacitors in a real 30-bus radial distribution network. Our approach can be extended to other networks also. Since one has to determine simultaneously the locations as well as the ratings of the capacitors, a two level strategy was devised. The ant colony algorithm maintained two pheromone tables, which ants used when generating solutions. The tables were updated simultaneously based on the lowest-cost solutions obtained. Our approach can be applied to other practical problems requiring complex decision making as well.

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